Thermalization process in weakly coupled systems

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New progress in HIC, Wuhan

Oct 5, 2015

In collaboration with:

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Sören Schlichting
Raju Venugopalan

Partially based on:

J. Berges, KB, S. Schlichting, and R. Venugopalan,

arXiv: 1508.03073 ;
PRL 114, 061601 (2015) ;

PRD 89, 074011 (2014) ;
PRD 89, 114007 (2014)

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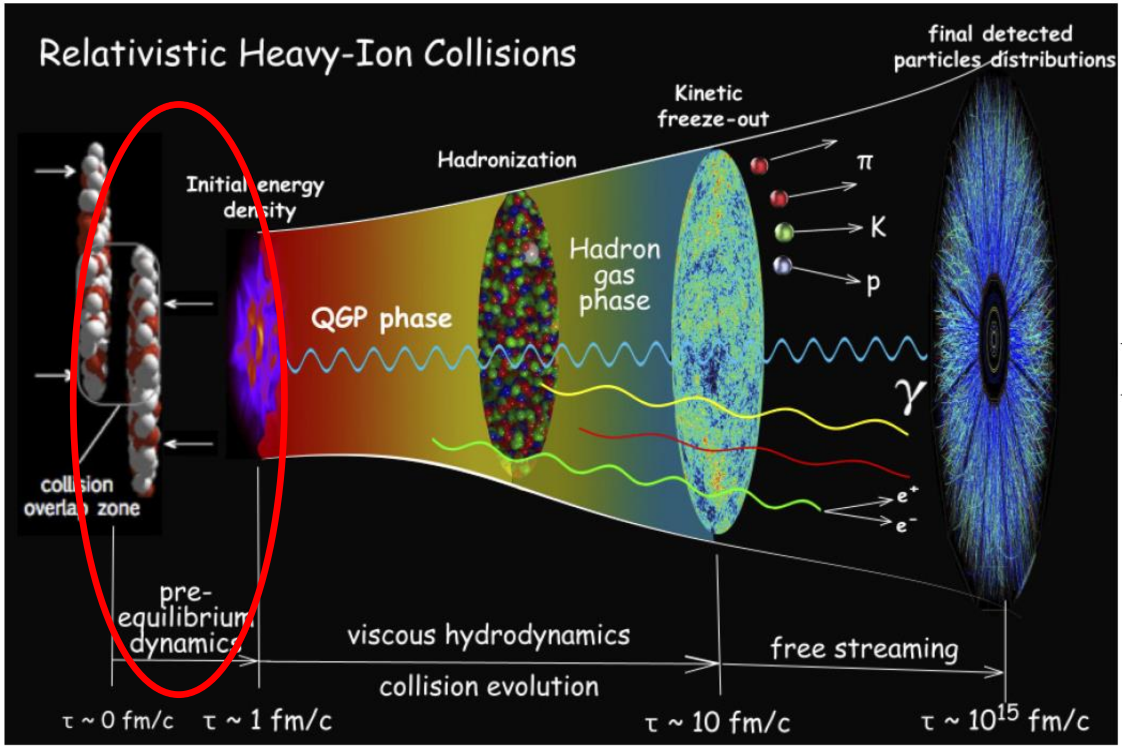
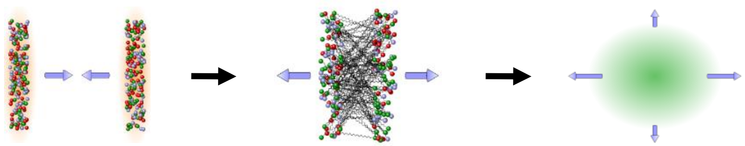
PART I: Thermalization process in heavy-ion collisions



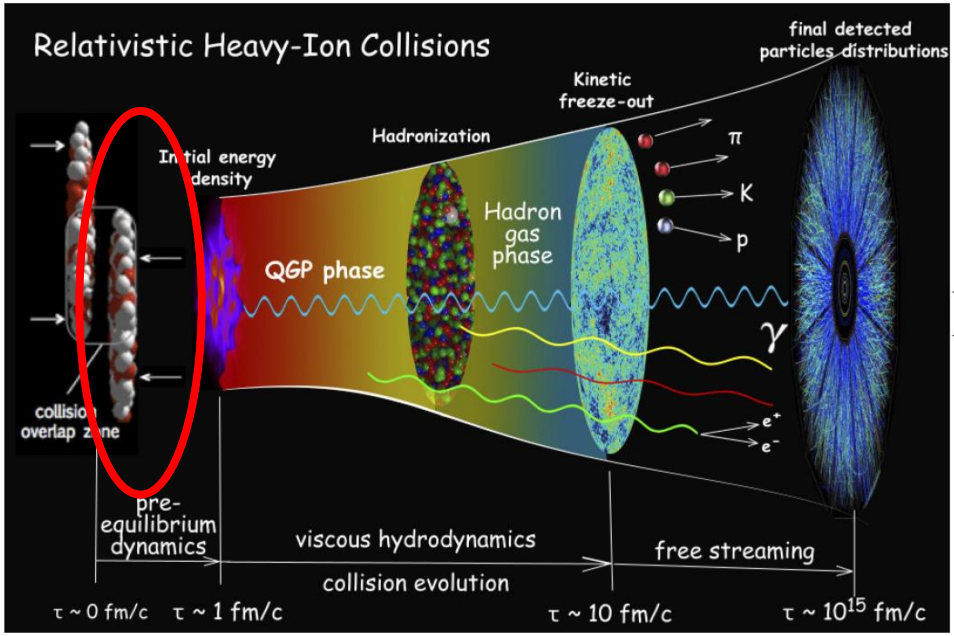
Little Bang by P. Sorensen and C. Shen

Thermalization in weakly coupled non-Abelian plasmas

Thermalization



Figs. by T. Epelbaum



Hydrodynamical simulations:

- Quick thermalization
- Nearly ideal fluid

High energy (weak coupling) limit in heavy-ion collisions $\alpha_s \ll 1$

(Microscopic theory: Quantum Chromodynamics, QCD)

In Bjorken coordinates:

$$\tau = \sqrt{t^2 - (x^3)^2}\,, \qquad \eta = \operatorname{artanh}\left(\frac{x^3}{t}\right)$$

Longitudinally expanding metric:

$$g_{\mu\nu}(\tau) = \operatorname{diag}\left(1, -1, -1, -\tau^2\right)$$

Thermalization in weakly coupled non-Abelian plasmas

Initial state: CGC, Glasma and classical fields

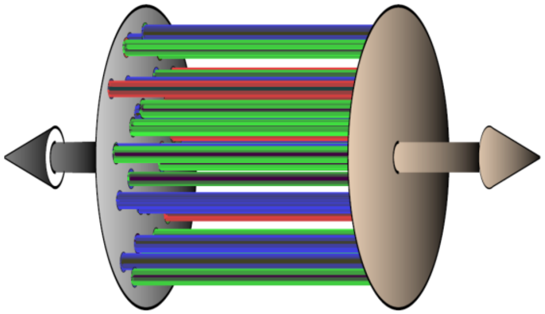
→ Talk by T. Lappi

Color glass condensate (CGC) effective theory:

Gelis, Iancu, Jalilian-Marian & Venugopalan, *Ann. Rev. Nucl. Part. Sci. 60, 463 (2010)*

Decomposition of hard (sources) and soft (plasma) partons; contains saturation scale Q_S

Glasma: Initial state of weakly coupled HIC at $Q_s \tau = 0^+$



Longitudinal chromo-electric and chromo-magnetic (classical) fields;
Boost-invariant at LO in g; 2+1 D classical equations of motion

Color charge densities of nuclei Gaussian distributed in transverse coordinates (McLerran-Venugopalan model)

- If functions of impact parameter → IP Glasma model, recent papers:
- We consider homogeneous and isotropic initial conditions

Schenke, Schlichting & Venugopalan, *PLB **747**, 76 (2015)*
Lappi, Schenke, Schlichting & Venugopalan, *arXiv: 1509.03499*

At NLO include vacuum fluctuations → explicit breaking of boost-invariance, instabilities

Thermalization in weakly coupled non-Abelian plasmas

Plasma instabilities at early times

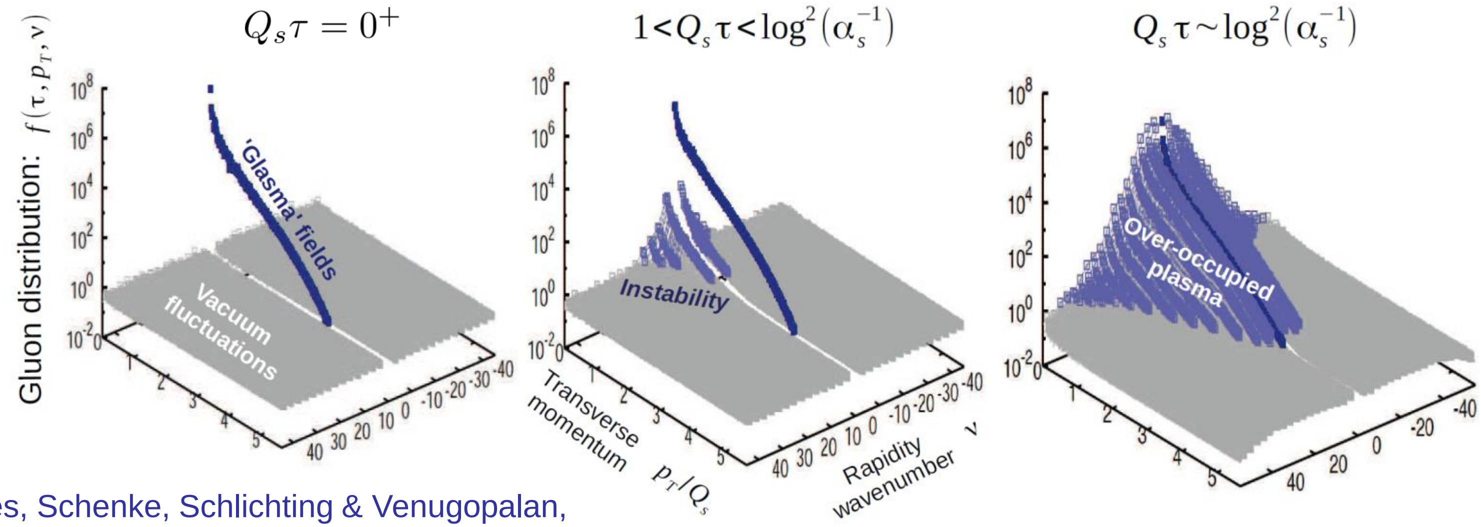
Literature: *Mrowczynski (1988); Arnold, Lenaghan & Moore (2003); Romathscke & Strickland (2003); Romatschke & Venugopalan (2006); Fukushima & Gelis (2012); Berges & Schlichting (2013); …*

Strongly correlated, mainly gluonic plasma created $f = \frac{d^3N}{d^2p_T dp_z} \sim \frac{1}{\alpha_s} \gg 1$

Classical background + vacuum fluctuations

Plasma instabilities

Over-occupied plasma



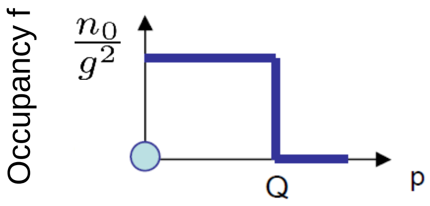
Berges, Schenke, Schlichting & Venugopalan, *Nucl. Phys.* ***A*** *931, 348 (2014)*

Thermalization in weakly coupled non-Abelian plasmas

Nonperturbative approach: Classical-statistical simulations

Typical initial conditions

Over-occupation IC



$$f(p_T, p_z, \tau_0) = \frac{n_0}{g^2} \Theta \left(Q - \sqrt{p_T^2 + (\xi_0 p_z)^2} \right)$$

Occupancy   Anisotropy

Weak couplings but highly correlated system

Weak coupling limit $g^2 \to 0$ while $g^2 f = const$

Fields follow *classical* evolution!

Observables averaged over (quantum) IC

Many field theory examples:

Micha & Tkachev ; Smit & Tranberg; Nowak, Sexty & Gasenzer ; Berges, KB, Schlichting & Venugopalan; Kurkela & Moore; …

Thermalization in weakly coupled non-Abelian plasmas

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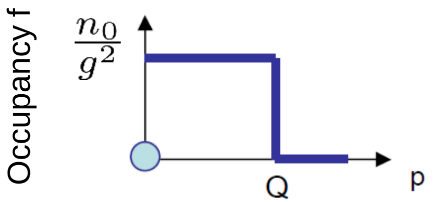
Over-occupation IC



Weak couplings but highly correlated system

Weak coupling limit $g^2 \to 0$ while $g^2 f = const$

Fields follow *classical* evolution!

Observables averaged over (quantum) IC

Classical equation of motion: $D_\mu F^{\mu\nu} = 0$

Fock-Schwinger gauge: $A_\tau = 0$

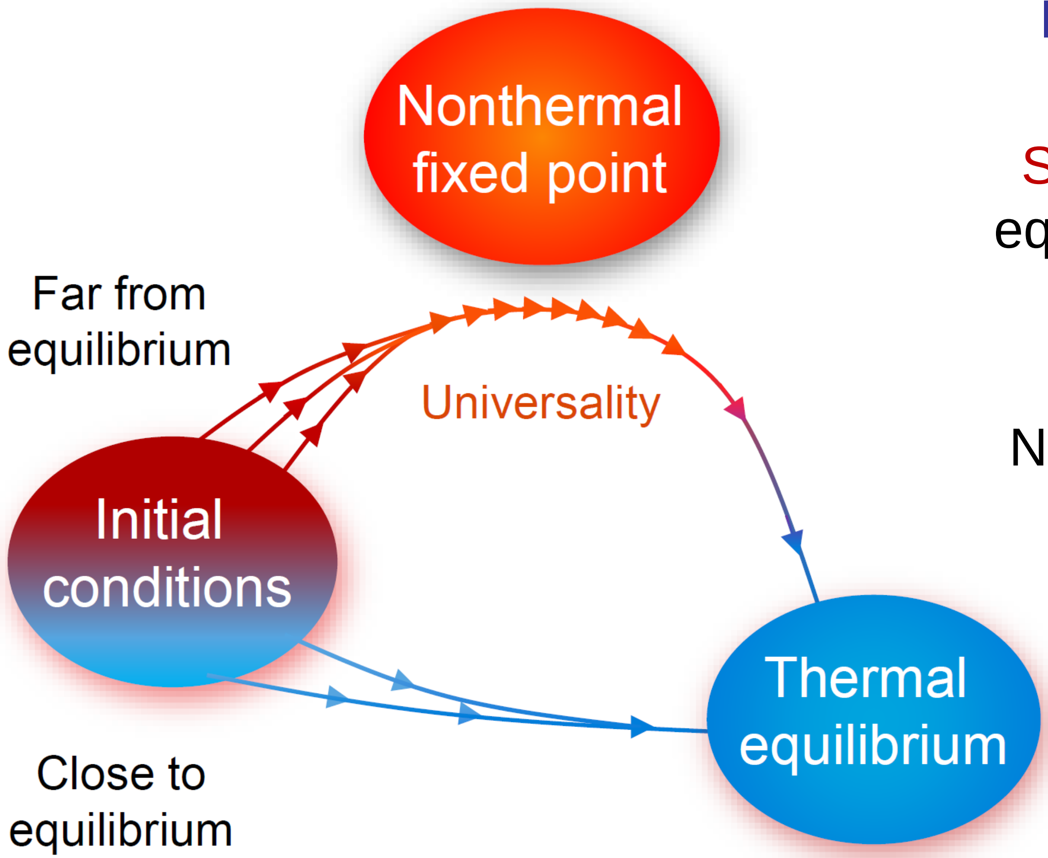
(Dynamics in link variables U and chromo-electric fields E)

Initialization: $A_\mu^a(\boldsymbol{x}, \tau_0) = \int_{\boldsymbol{p}} \sqrt{f(\boldsymbol{p}, \tau_0)} \left(c_{a,\lambda}(\boldsymbol{p}) \; \xi_\mu^{(\lambda)}(\boldsymbol{p}, (\tau_0)) \; e^{i\boldsymbol{p}\boldsymbol{x}} + c.c. \right)$

Gaussian distributed complex random numbers

Mode functions: solutions of free equations of motion

Thermalization in weakly coupled non-Abelian plasmas



In general for field theories:

Strong correlations: Far-from-equilibrium initial conditions (IC)

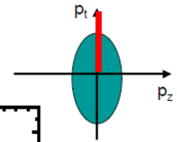
Nonthermal fixed point (NTFP)

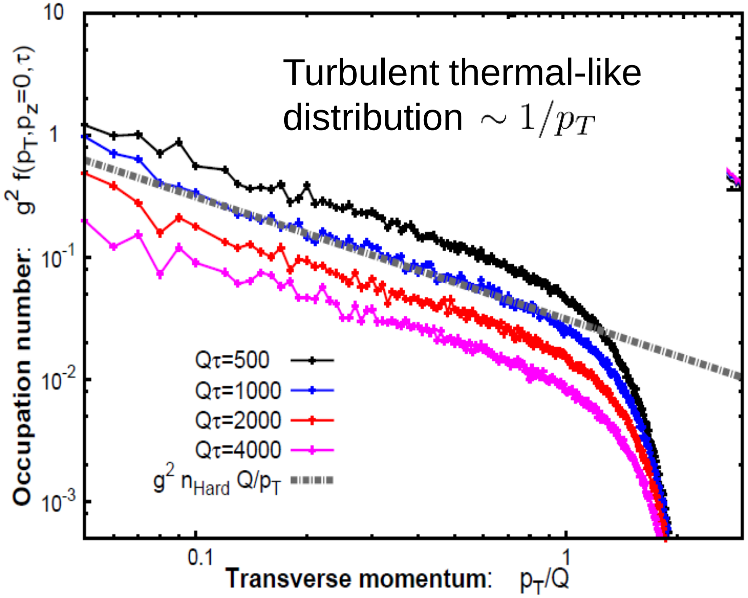
- ✓ Partial memory loss
- ✓ Time scale independence
- ✓ Self-similar dynamics

$$f(p_T, p_z, \tau) = \tau^{\alpha} f_S\left(\tau^{\beta} p_T, \tau^{\gamma} p_z\right)$$

Thermalization in weakly coupled non-Abelian plasmas

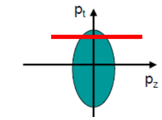
Transverse shape of distribution

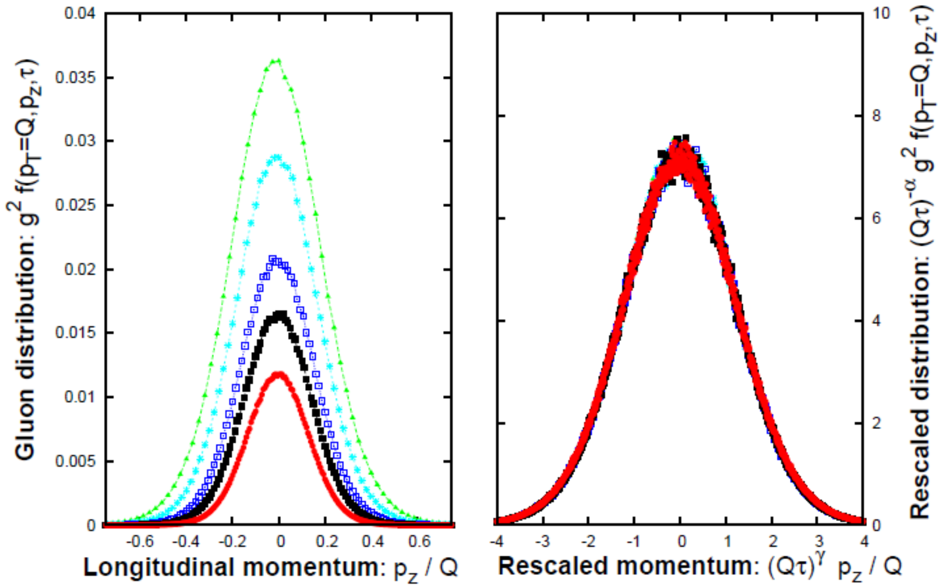




Berges, KB, Schlichting & Venugopalan,
PRD 89, 074011 (2014)

Longitudinal shape of distribution





Berges, KB, Schlichting & Venugopalan,
PRD 89, 114007 (2014)

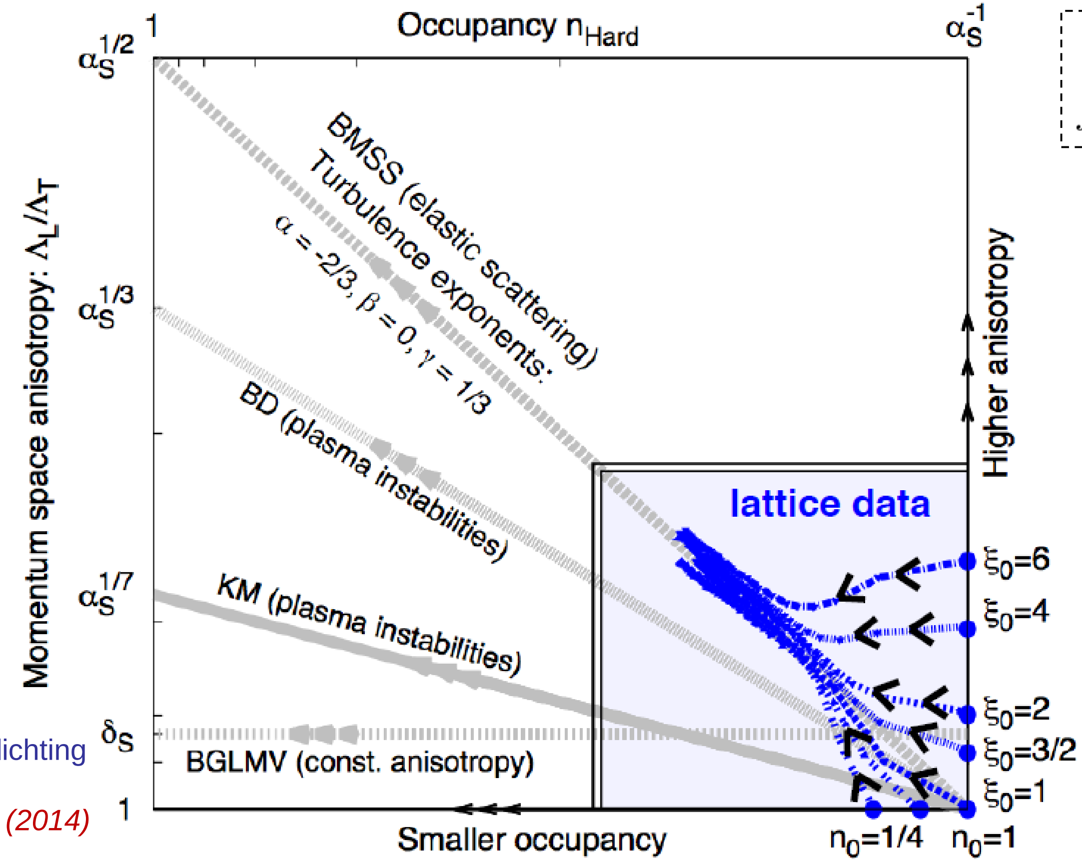
Self-similar evolution

$$f(p_t, p_z, \tau) = \tau^{\alpha} f_s(\tau^{\beta} p_t, \tau^{\gamma} p_z)$$

Scaling exponents

$$\begin{aligned} \alpha &\simeq -2/3 \\ \beta &\simeq 0 \\ \gamma &\simeq 1/3 \end{aligned}$$

Thermalization in weakly coupled non-Abelian plasmas



Berges, KB, Schlichting & Venugopalan, *PRD 89, 074011 (2014)*

Reminder: Self-similar evolution

$$f(p_T, p_z, t) = t^\alpha f_S(t^\beta p_T, t^\gamma p_z)$$

Simulations approach nonthermal fixed point with scaling exponents

$$\alpha = -2/3 \quad \beta = 0 \quad \gamma = 1/3$$

Nonthermal fixed point matches the BMSS scenario!

Thermalization scenarios

Baier, Mueller, Schiff, Son (BMSS), (2001)

Bodeker (BD), (2005)

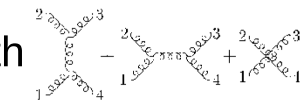
Kurkela, Moore (KM), (2011)

Blaizot, Gelis, Liao, McLerran, Venugopalan (BGLMV), (2012)

Thermalization in weakly coupled non-Abelian plasmas

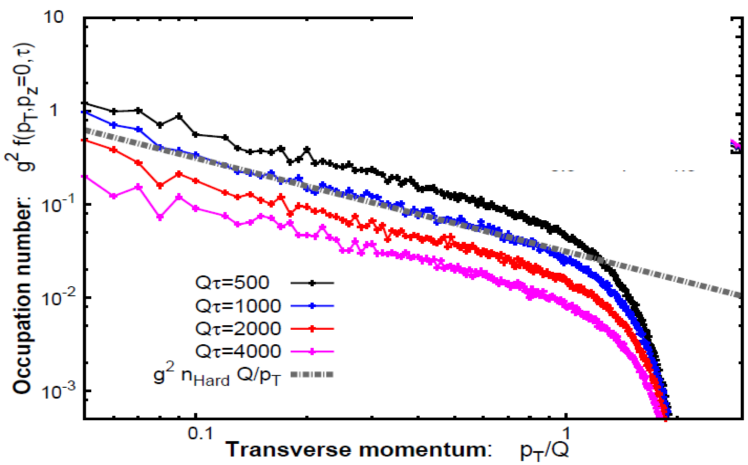
‚Bottom-up‘ thermalization scenario

Baier, Mueller, Schiff & Son, *PLB 502, 51 (2001)*

Description within effective kinetic theory (AMY) with  and  processes

Arnold, Moore & Yaffe, *JHEP 0301, 030 (2003)*

Overoccupied plasma $f \gg 1$

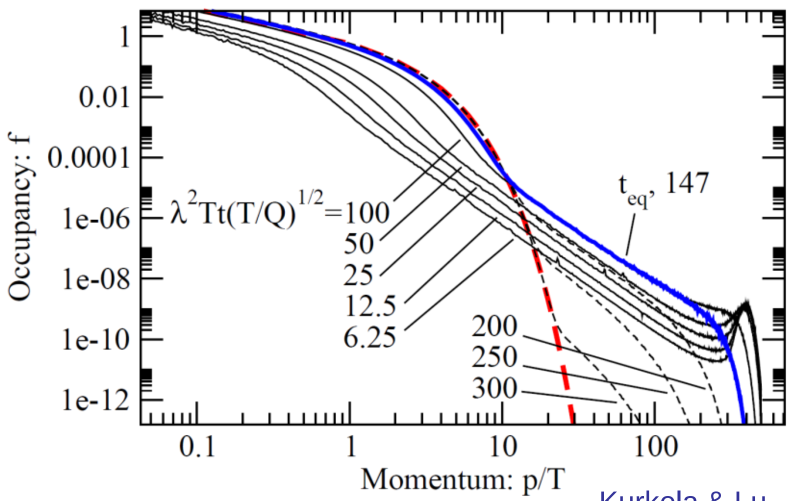


BBSV, *PRD **89**, 074011 (2014)*

Classical attractor

Occupancy ↘ , anisotropy ↗

Underoccupied plasma $f \ll 1$



Kurkela & Lu, *PRL **113**, 182301 (2014)*

Radiational breakup, wave turbulence & thermal IR

→ Talk by Y. Mehtar-Tani

Blaizot, Iancu & Mehtar-Tani, *PRL **111**, 052001 (2013)*

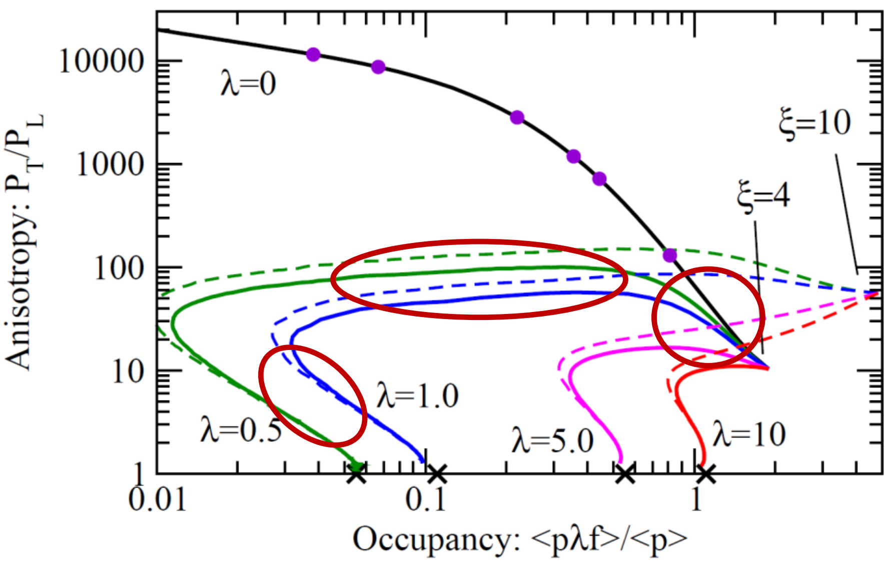
Thermalization in weakly coupled non-Abelian plasmas

‚Bottom-up‘ picture and onset of hydrodynamics

Kurkela & Zhou, *1506.06647*

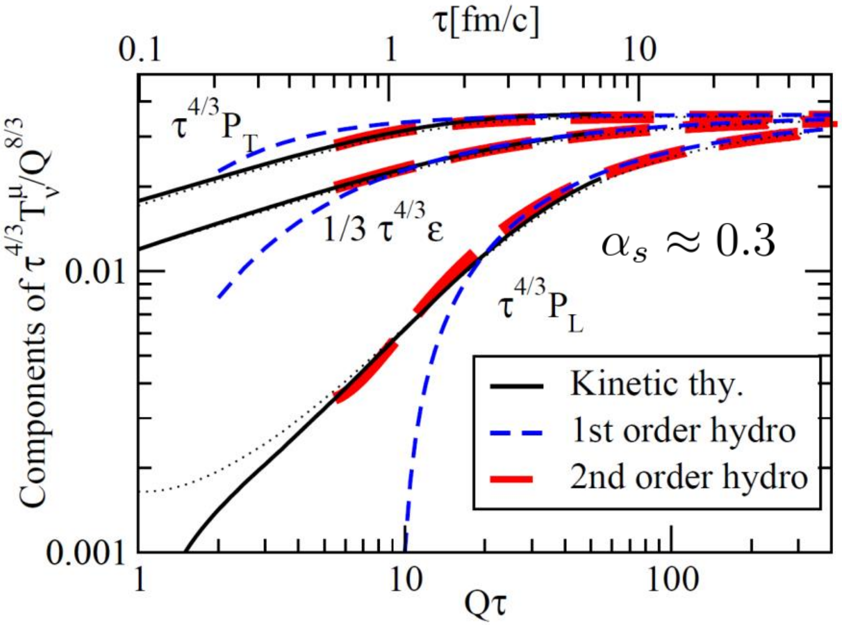
Full ‚bottom-up‘ evolution

($\lambda = 4\pi\alpha_s N_c$ extrapolated to moderate values = 5, 10)



3 stages: i) classical scaling; ii) anisotropy freezes; iii) radiational breakup

Onset of hydrodyncamics



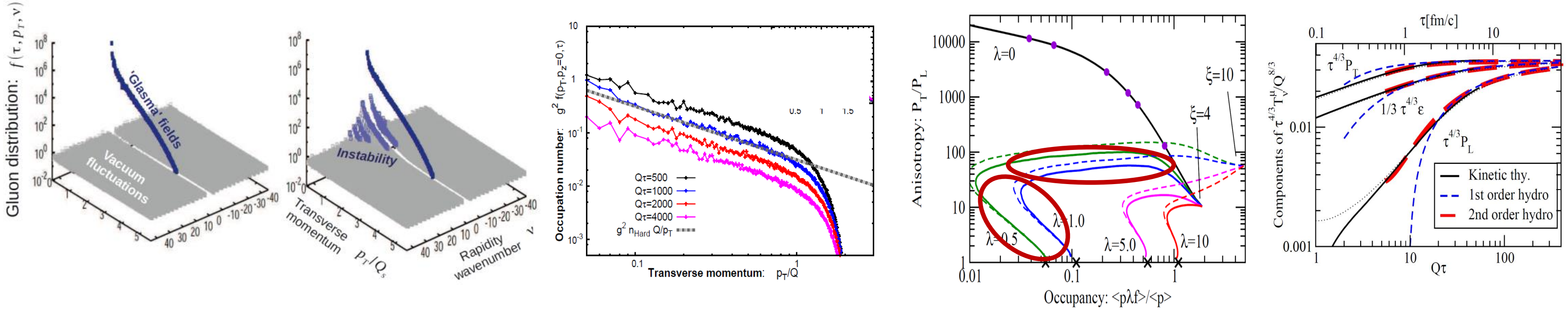
Hydrodynamics already at $\tau \lesssim 1 fm/c$!

Thermalization in weakly coupled non-Abelian plasmas

Summary of part I:

The entire thermalization process

$Q_s\tau = 0^+$ | $Q_s\tau = \ln^2(\alpha_s^{-1})$ | $Q_s\tau = \alpha_s^{-3/2}$ | $Q_s\tau = \alpha_s^{-13/5}$



Glasma; | Instabilities; | Classical NTFP; | Constant anisotropy, Radiational breakup; | Hydrodynamics, Thermalization

‚Bottom-up' scenario based on AMY kinetic theory;

Outlook: What changes with NLO contributions in g?

Ghiglieri, Moore & Teaney,
arXiv: 1502.03730
arXiv: 1509.07773

Thermalization in weakly coupled non-Abelian plasmas

Summary of part I:

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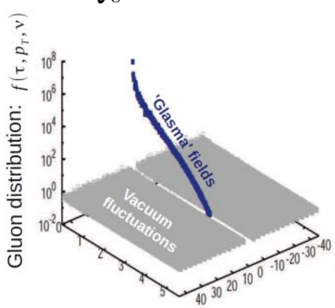

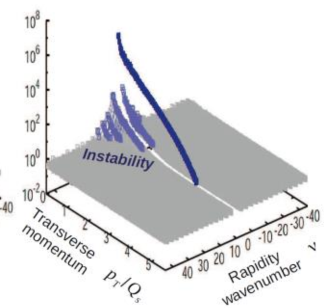

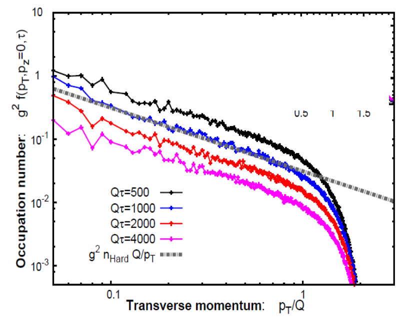

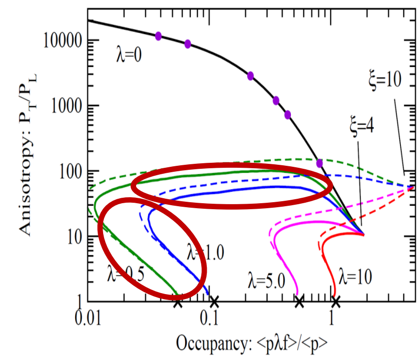

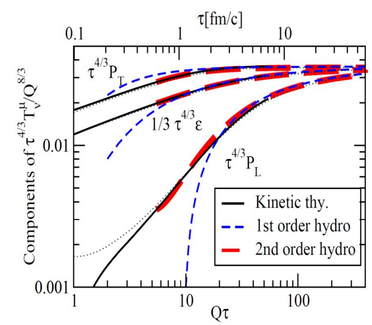

Glasma; | Instabilities; | Classical NTFP; | Constant anisotropy, Radiational breakup; | Hydrodynamics, Thermalization

Next: closer look

Part II: Universality classes and remaining puzzles

Massless scalar field theory (O(N))

$$S = \int d\tau d^2 x_T d\eta \; \tau \left(\frac{g^{\mu\nu}}{2} (\partial_\mu \varphi_a)(\partial_\nu \varphi_a) - \frac{\lambda}{4! N} (\varphi_a \varphi_a)^2 \right)$$

Non-Abelian gauge theory (SU(2))

$$S = \int d\tau d^2 x_T d\eta \; \tau \; F^a_{\mu\nu} F^{a,\mu\nu}$$

Compare nonthermal fixed points in longitudinally expanding geometry

J. Berges, KB, S. Schlichting and R. Venugopalan:

PRL 114, 061601 (2015) ; arXiv: 1508.03073 ;

Universality classes and remaining puzzles

What we are after: Scaling regions and universality classes

Scaling region (close to a nonthermal fixed point)

- *Self-similar evolution* of distribution function f (→ slow dynamics, memory loss)

$$f(p_T, p_z, \tau) = \tau^{\alpha} f_S\left(\tau^{\beta} p_T, \tau^{\gamma} p_z\right)$$

with scaling behavior of typical scales $f \sim \tau^{\alpha}$, $p_T \sim \tau^{-\beta}$, $p_z \sim \tau^{-\gamma}$

Universality classes and remaining puzzles

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Classification: universality classes far from equilibrium

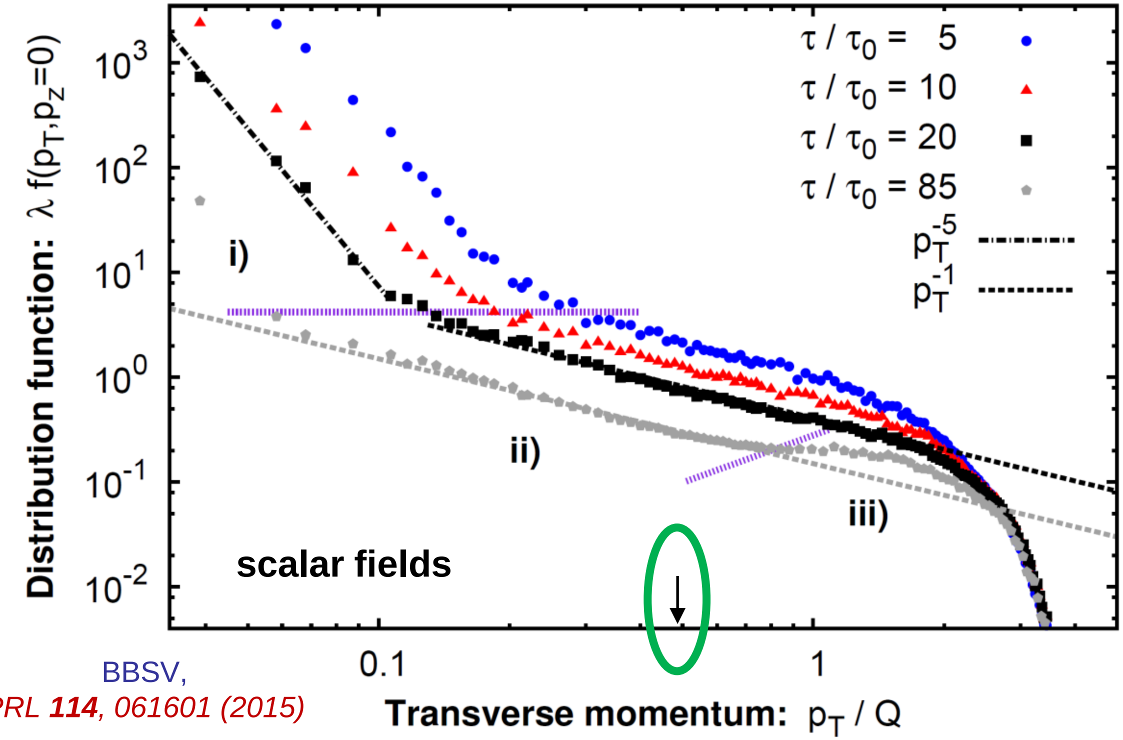
- Scaling regions, described by their exponents α, β, γ and the scaling function $f_S(x,y)$, may be classified in universality classes if compared between different microscopic theories

Analogy:

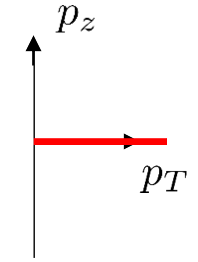
NTFP	Close to 2nd order PT
Time scale τ	Temperature scale $\tau = (T-T_c)/T_c$
Self-similar evolution	Critical slowing down
Scaling exponents & function	Critical exponents & surface

Universality classes and remaining puzzles

Scalar nonthermal attractor: Different scaling regions i), ii) and iii)



BBSV,
*PRL **114**, 061601 (2015)*



Regions ii) and iii):

Local conservation of particle number and energy density in p_T:

$$\tau\, dn/dp_T\,,\quad \tau\, d\epsilon/dp_T$$

are time-independent

Effectively no flux in p_T!

Reminder: *Self-similar evolution*

$$f(p_T, p_z, \tau) = \tau^{\alpha} f_S(\tau^{\beta} p_T, \tau^{\gamma} p_z)$$

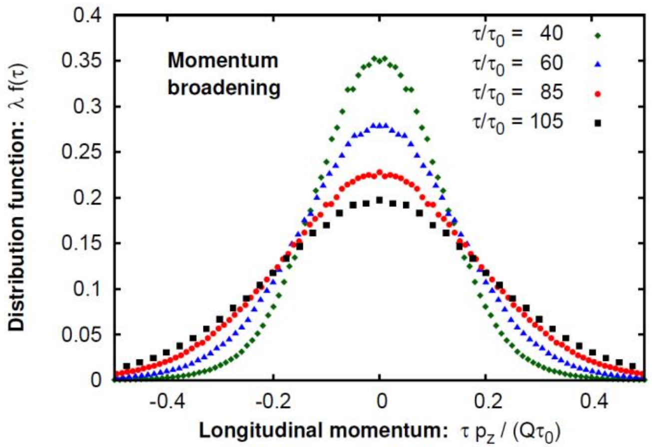
implies

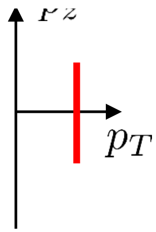
$$\begin{aligned} \alpha - \gamma &= -1 \\ \beta &= 0 \end{aligned}$$

Universality classes and remaining puzzles

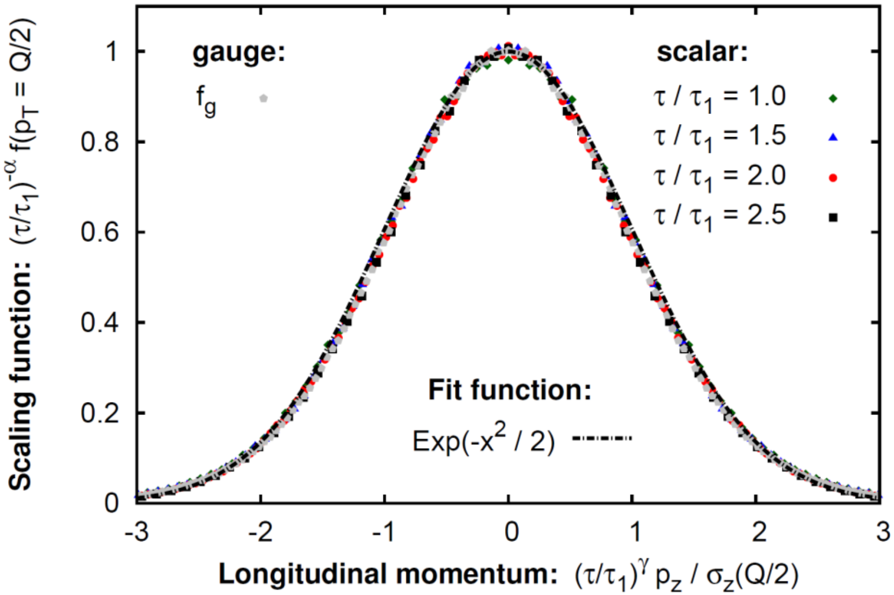
Longitudinal dynamics in scaling region ii)

BBSV,
*PRL **114**, 061601 (2015)*





Rescaling with



$$\begin{aligned} \alpha &= -2/3 \\ \gamma &= 1/3 \end{aligned}$$

leads to time-independent distribution!
Well described by Gaussian shape.

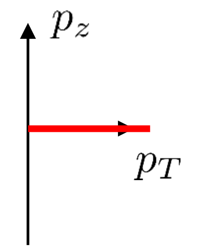
Reminder: *Self-similar evolution*

$$f(p_T, p_z, \tau) = \tau^{\alpha} f_S(\tau^{\beta} p_T, \tau^{\gamma} p_z)$$

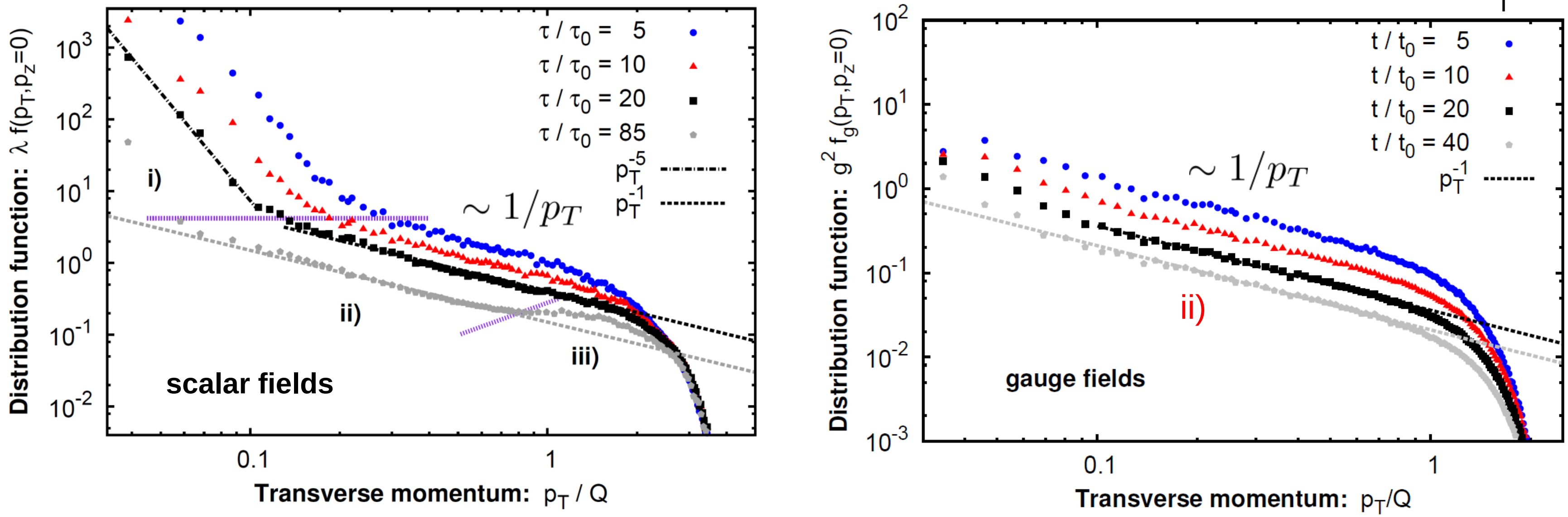
Same form as for gauge theory! (also exponents)

Common universality class!

Universality classes and remaining puzzles



Where is common scaling region?



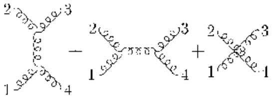
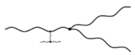
- Scaling range ii) given by $\lambda f \sim \frac{\tau^{-2/3}}{p_T} e^{-p_z^2/2\sigma_z^2}$ with $\sigma_z^2 = \frac{\int dp_z\, p_z^2 f}{\int dp_z f} \sim \tau^{-2/3}$
- Exponents and structure insensitive to initial conditions (memory loss)

J. Berges, KB, S. Schlichting, and R. Venugopalan: *PRD 89, 074011 + 114007 (2014) ; arXiv:1508.03073*

Universality classes and remaining puzzles

Puzzles

Gauge theory:

✓ Gauge theories for $p \gtrsim m_D$ and $f \gg 1/\alpha_S$ well described by effective kinetic theory (AMY) including  

Arnold, Moore & Yaffe, JHEP 0301, 030 (2003)

Baier, Mueller, Schiff & Son, PLB 502, 51 (2001)

Berges, KB, Schlichting & Venugopalan, PRD 89, 074011 (2014)

Kurkela & Zhou, 1506.06647

BUT: Thermalization scenarios suggested influence from IR (plasma instabilities, condensates, …) → **no influence from IR? Why?**

Bodeker (BD), (2005)

Kurkela, Moore (KM), (2011)

Blaizot, Gelis, Liao, McLerran, Venugopalan (BGLMV), (2012)

Universality classes and remaining puzzles

Puzzles

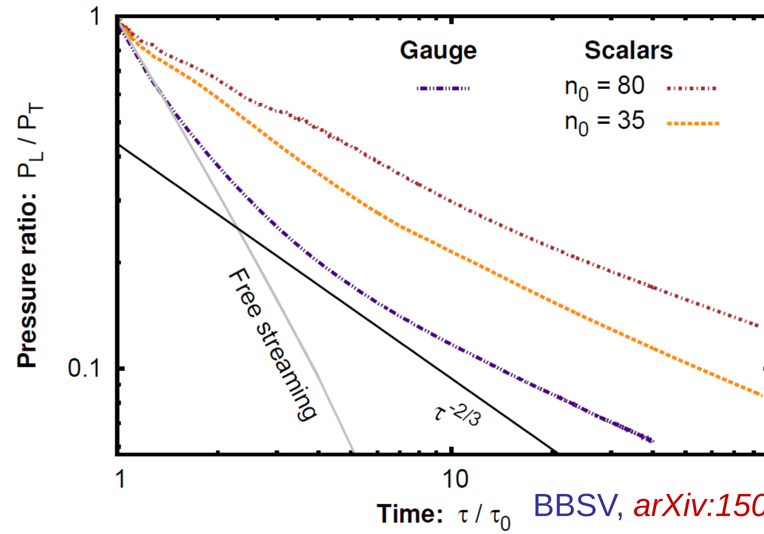
Pressure ratio:

Parametrically:

$$\frac{P_L}{P_T} \overset{\text{kinetic theory}}{\sim} \frac{\int d^3p\, p_z^2/\omega f}{\int d^3p\, p_T^2/\omega f} \overset{\text{late times}}{\sim} (Q\tau)^{-2/3}$$

Discrepancies because of IR?

In scalar theory nontrivial IR dynamics!



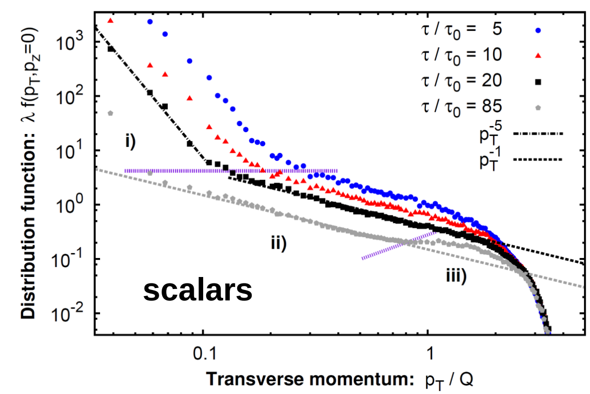
BBSV, *arXiv:1508.03073*

Scalar theory:

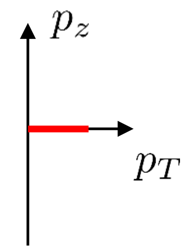
How can region ii) be microscopically understood?

How important is soft region for it?

How does Bose condensation emerge?



Universality classes and remaining puzzles



Self-similar evolution in IR

Reminder: *Self-similar evolution*

$$f(p_T, p_z, \tau) = \tau^{\alpha} f_S(\tau^{\beta} p_T, \tau^{\gamma} p_z)$$

Exponents:

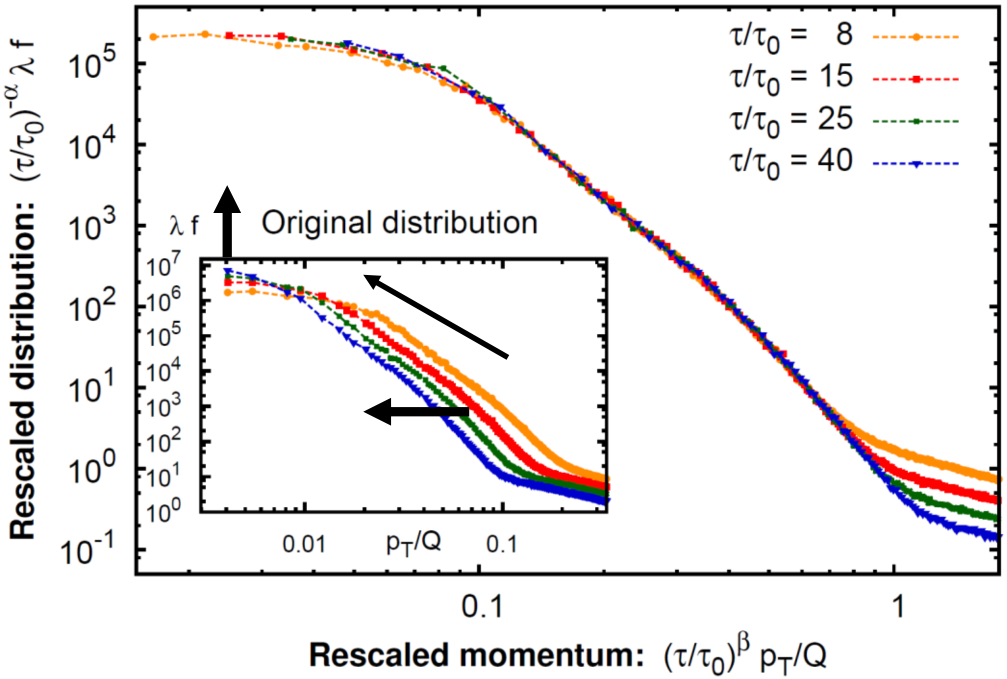
$$\alpha = 1$$

$$\beta = 2/3$$

From isotropic evolution $\gamma = \beta$

Particle number conservation

$$n \sim \int d^3p f \sim 1/\tau$$



Berges, KB, Schlichting & Venugopalan,
arXiv:1508.03073

Physical picture: Inverse particle cascade to IR

Bose condensation far from equilibrium!

J. Berges & D. Sexty,
PRL 108 (2012), 161601

Pinerio Orioli, KB & Berges,
PRD 92, 025041 (2015)

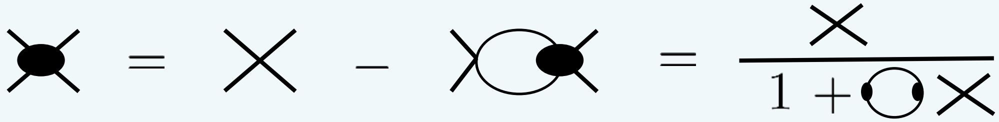
Universality classes and remaining puzzles

Kinetic approach: Vertex-resummed kinetic theory

IR dynamics contains very high occupancy $1/\lambda \gg f$.

Possible solution: vertex-resummed kinetic theory based on 2PI 1/N expansion to NLO

Loosely diagrammatically written:



Rigorously:

Berges & Sexty,
PRD 83, 085004 (2011)

Pinerio Orioli, KB & Berges,
PRD 92, 025041 (2015)

The is kinetic theory explains scaling properties in expanding and also nonexpanding scalar systems.

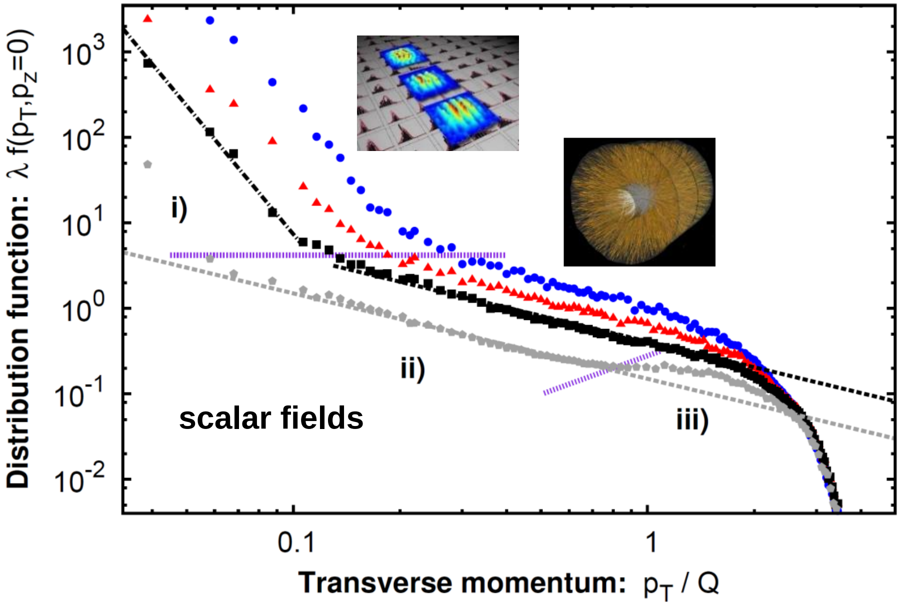
Pinerio Orioli, KB & Berges,
PRD 92, 025041 (2015)

Berges, KB, Schlichting & Venugopalan,
arXiv:1508.03073

What IR dynamics exists in gauge theories?
How to describe it?

Universality classes and remaining puzzles

The entire attractor in longitudinally expanding scalars



Reminder: *Self-similar evolution*

$$f(p_T, p_z, \tau) = \tau^{\alpha} f_S(\tau^{\beta} p_T, \tau^{\gamma} p_z)$$

Berges, KB, Schlichting & Venugopalan,
arXiv:1508.03073

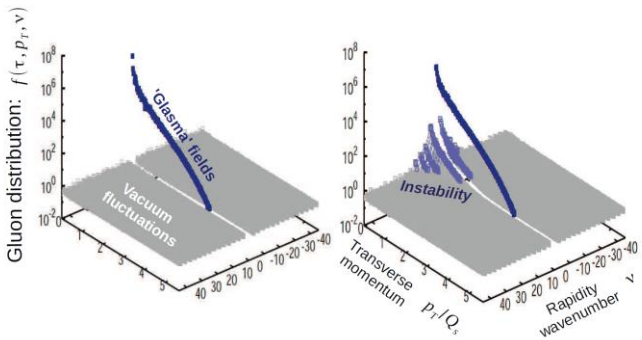
Fixed points	α	β	γ	λf_S
i)	1	$2/3$	$2/3$	$\left((p/b)^{-1/2} + (p/b)^{-5}\right)^{-1}$
ii)	$-2/3$	0	$1/3$	$p_T^{-1}\ e^{-p_z^2/2\sigma_z^2}$
iii)	$-1/2$	0	$1/2$	$\mathrm{sech}(p_z/\sigma_z)$

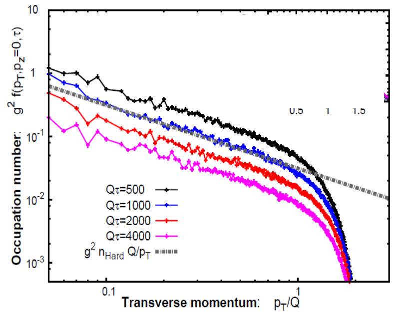
Conclusion:

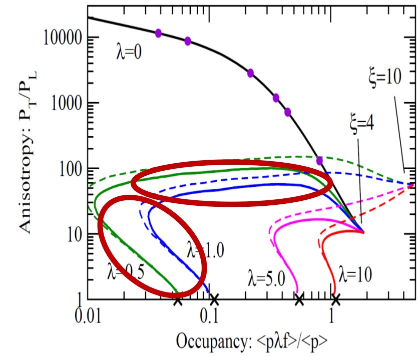
- Many aspects of thermalization process in heavy-ion collisions at weak couplings understood; early onset of hydrodynamics $\tau \lesssim 1 fm/c$:
- Glasma → Instabilities → ‚Bottom-up‘ → Hydrodynamics, thermalization
- Nonthermal scaling regions may be classified in universality classes; universality between scalar and gauge theories in expanding geometry found

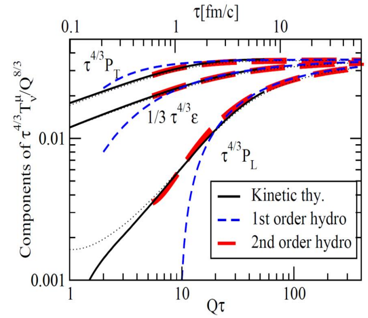
Outlook:

- IR region needs to be better understood in gauge theories.
- What changes when i) quarks, ii) NLO contribution to AMY kintic theory or iii) structure in spatial transverse plane are included?
- How comes that expanding scalars show same scaling region as gauge theory?

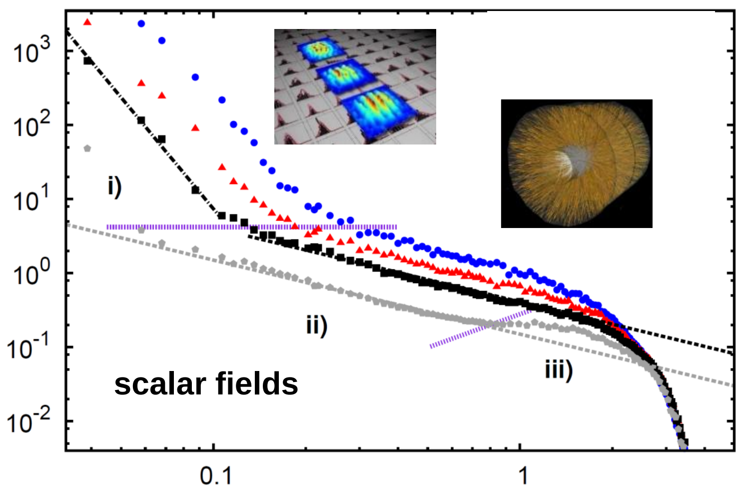


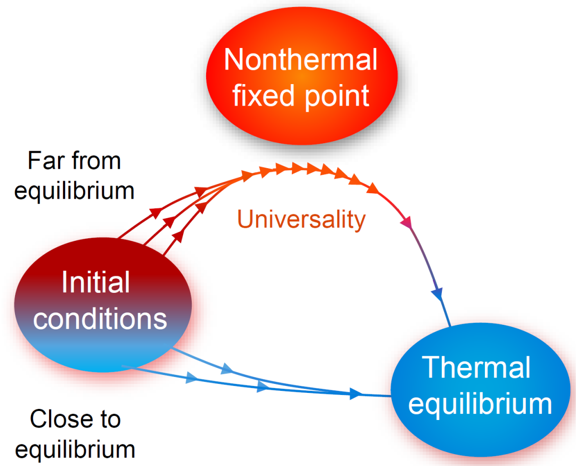






Thank you for your attention!



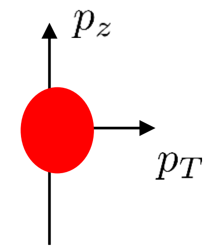


BACKUP SLIDES

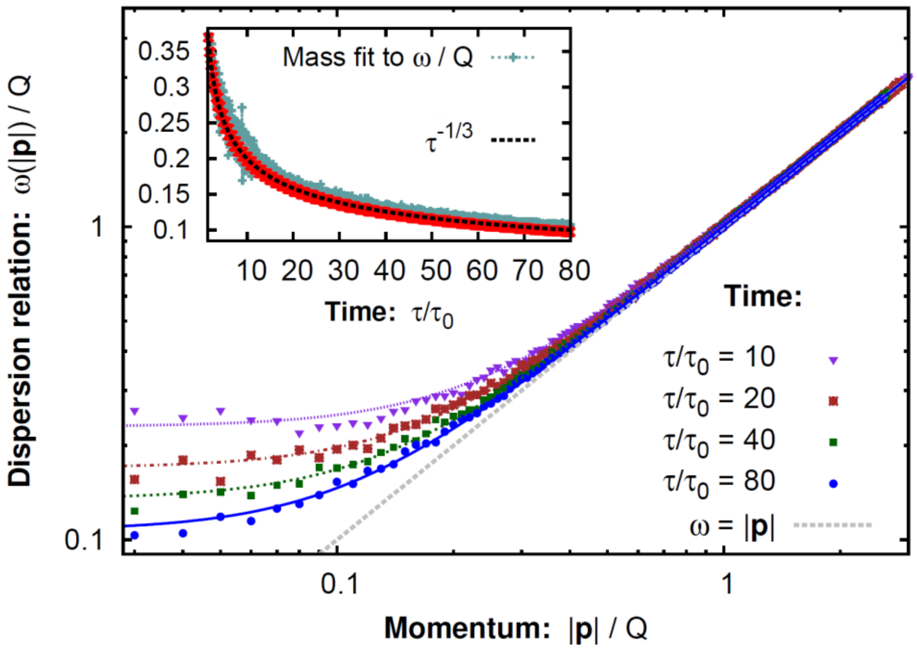
Universality classes and remaining puzzles

Scalar fields infrared scaling region: i)

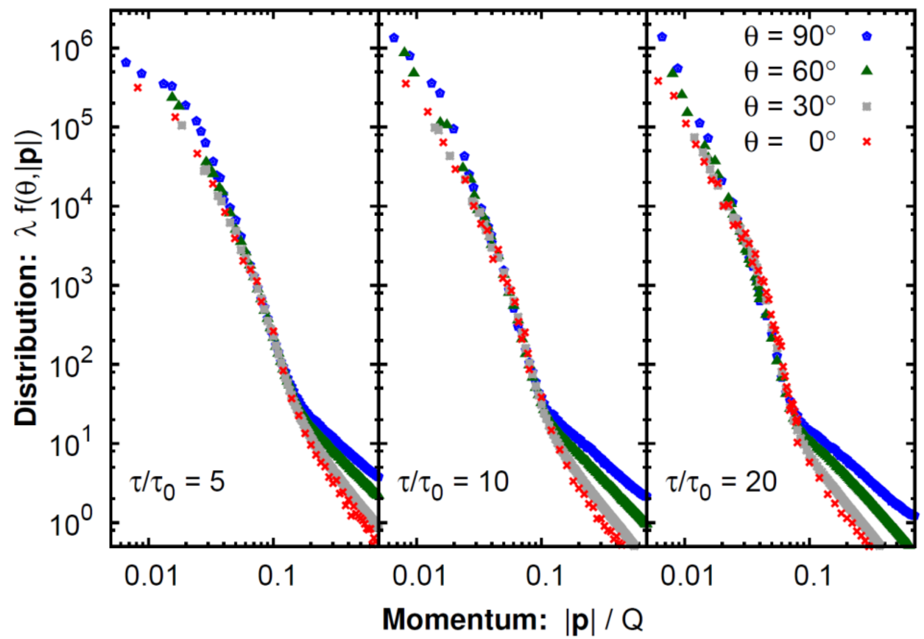
Expanding



Dynamically generated *mass*



(approx.) *isotropic* distribution



Dispersion relation fit: $\sqrt{m^2 + p^2}$

$$m(\tau) \sim \tau^{-1/3}$$

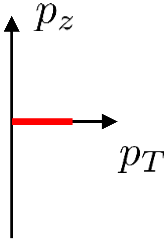
All momenta in i) below mass! $p \lesssim m$

Effectively nonrelativistic infrared region

Universality classes and remaining puzzles

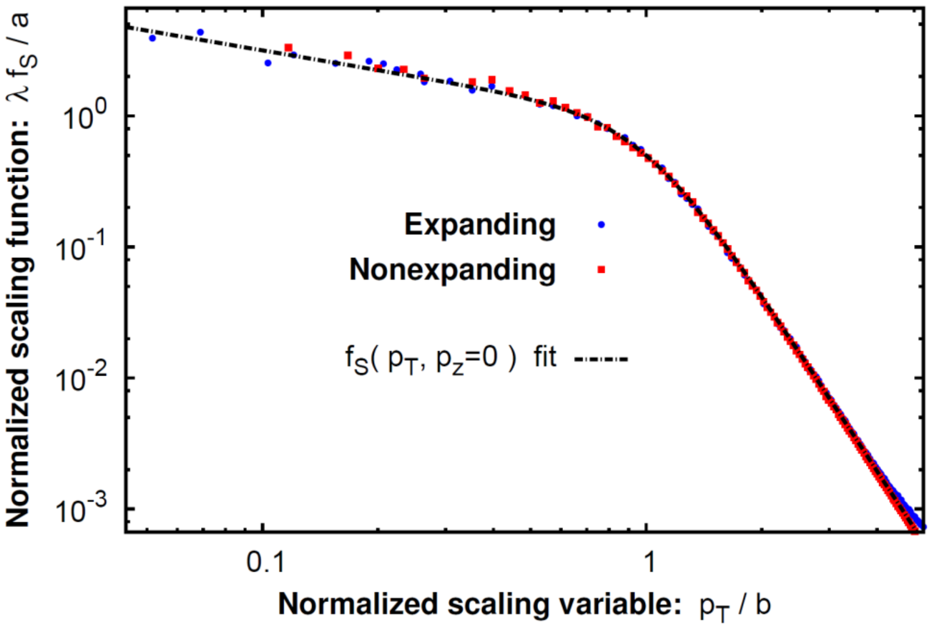
Universal scaling function

Expanding



$$\lambda f_S = \frac{a}{(|\mathbf{p}|/b)^{\kappa_<} + (|\mathbf{p}|/b)^{\kappa_>}}$$

with $\kappa_< \simeq 0.5\,,\ \kappa_> \simeq 4.5 - 5$



Same function for **nonexpanding**:

- *O(N) scalar theories* (N > 1)
- *Nonrelativistic scalars*

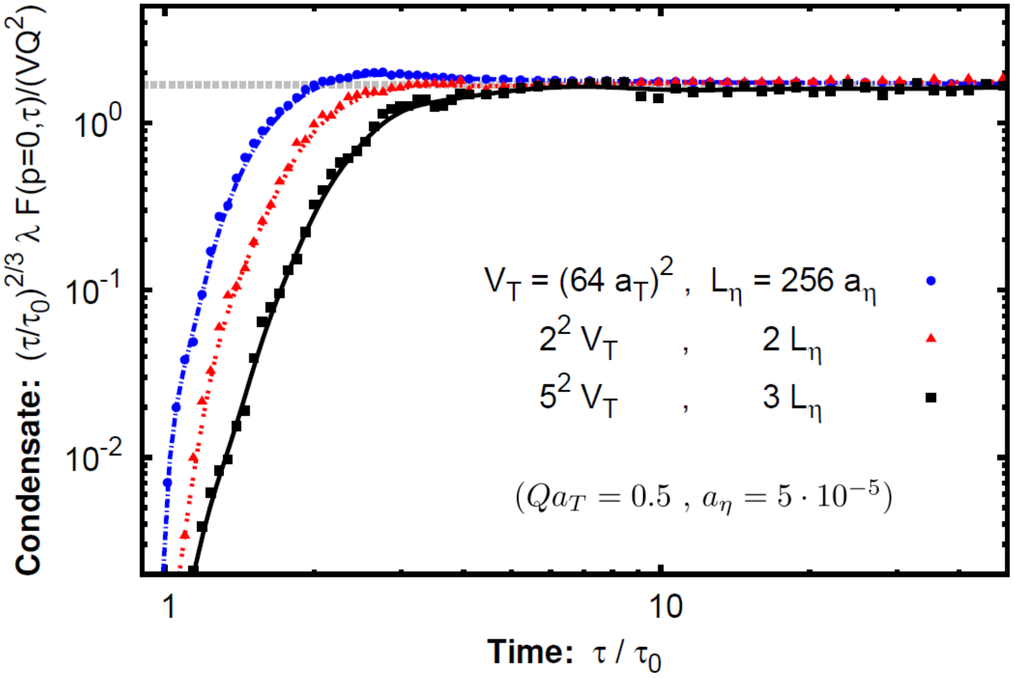
Nonexpanding scalars

Reminder: *Self-similar evolution*

$$f(p_T, p_z, \tau) = \tau^\alpha f_S(\tau^\beta p_T, \tau^\gamma p_z)$$

Longitudinally expanding scalar fields

Bose-Einstein condensation



If global Bose condensate exists, then the scalar field zero mode should scale as

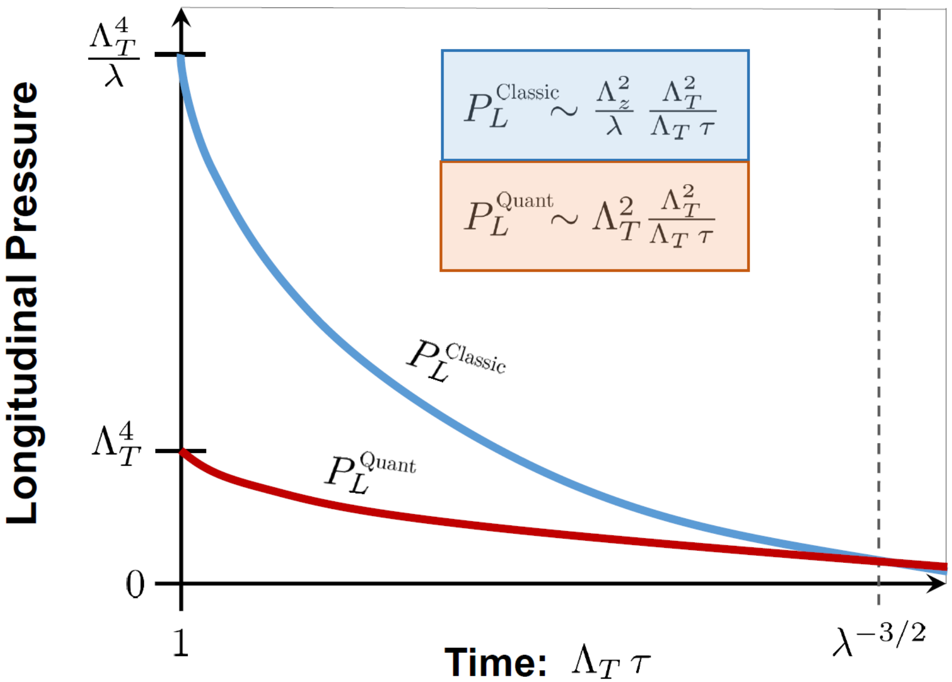
$$F(p=0) \sim \phi^2 \delta^{(3)}(\boldsymbol{p}) \sim \phi^2 V$$

J. Berges & D. Sexty, *PRL 108 (2012), 161601*

Bose-Einstein condensation far from equilibrium observed!

Universality classes and remaining puzzles

Break-down of classical dynamics in scalar theory



Classical pressure (intermediate p_T)

$$P_L^{\text{Classic}} \sim \int d^3p\, \frac{p_z^2}{p_T} f(p_T, p_z)$$

Quantum part: $\sigma_{\text{Large-angle}} \sim \lambda^2/\Lambda_T^2$

$$\frac{dN^{\text{Coll}}_{\text{Large-angle}}}{d\tau} \sim \sigma_{\text{Large-angle}} N_{\text{hard}}(\tau)$$

$$N_{\text{Large-angle}}(\tau) \sim N_{\text{hard}}(\tau) \frac{dN^{\text{Coll}}_{\text{Large-angle}}}{d\tau}\tau$$

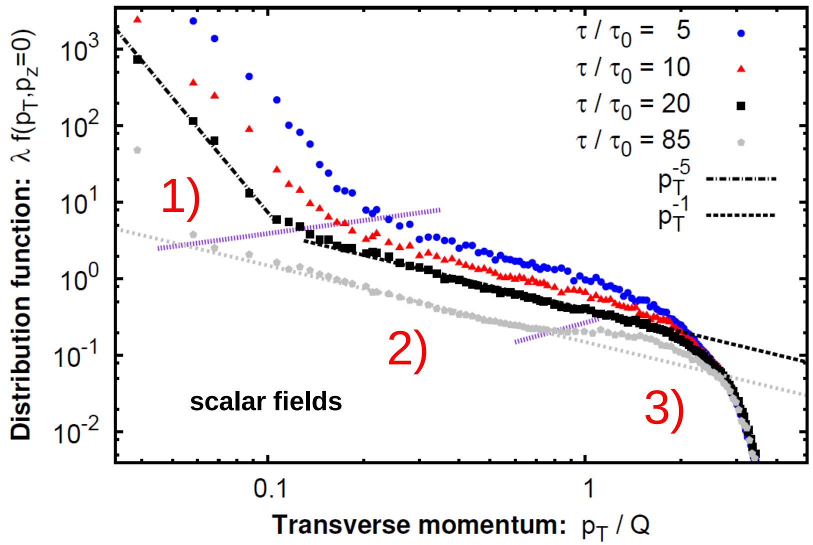
$$P_L^{\text{Quant}}(\tau) \sim N_{\text{Large-angle}}(\tau)\Lambda_T$$

Break-down at same time when classical approximation breaks down: $f \sim 1$

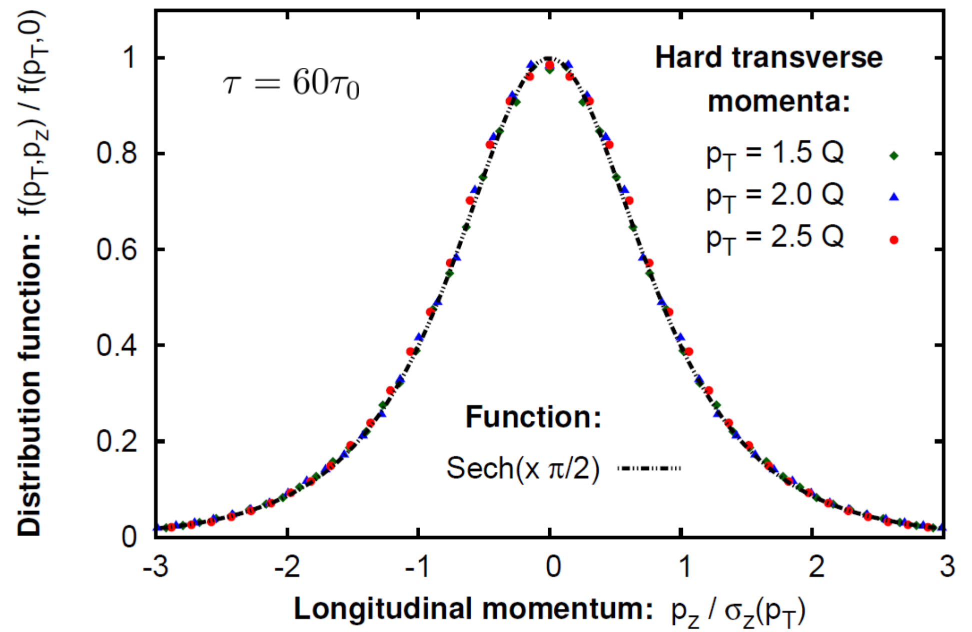
No new constraint!

Longitudinally expanding systems

Scalars fields hard-momentum fixed-point: Inertial range 3)



Longitudinal distribution at hard p_T



At late times a non-thermal fixed-point emerges at *large momenta*.

It has a hyperbolic secant shape, which has a broader tail than the Gaussian function.

Sign for large angle scatterings? $\times$ $(2 \leftrightarrow 2)$

Longitudinally expanding systems

Scalars fields hard-momentum fixed-point: Inertial range 3)

Self-similar evolution at large p_T

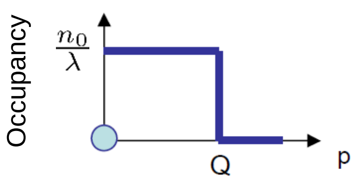
$$f(p_T, p_z, \tau) = \tau^{\alpha'} f'_S(p_T, \tau^{\gamma'} p_z)$$

Longitudinal hard scale

$$\Lambda_L^2(\tau) \approx \frac{\int d^2p_T \int dp_z\ p_z^2\, \omega(\mathbf{p})\ f(p_T, p_z, \tau)}{\int d^2p_T \int dp_z\ \omega(\mathbf{p})\ f(p_T, p_z, t)}$$

$$\sim \tau^{-2\gamma'}$$

Independent of initial conditions!



$$f(p_T, p_z, \tau_0) = \frac{n_0}{\lambda} \Theta\left(Q - \sqrt{p_T^2 + (\xi_0 p_z)^2}\right)$$

Logarithmic slope of hard scale